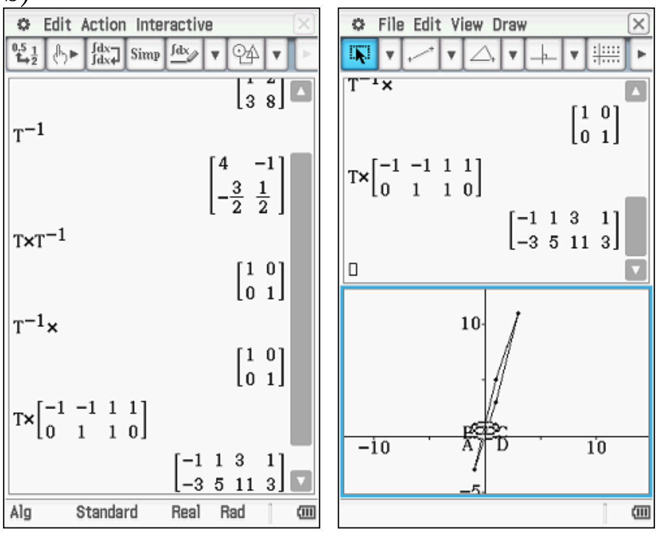
Activity 33 Inverse transformations

1. For T an $m \times n$ matrix:
   TB=I means B is an $n \times m$ matrix and
   I is an $m \times m$ Identity matrix.
   AT=I means that A is an $n \times m$ matrix and I is an $n \times n$ matrix.
   Thus AT and BT are different sizes for m not equal to n.

2.
   b)



3. $A(-1,-2), B(1,3), C(4,-1)$ $\left(T^{-1}\begin{bmatrix}-5 & 7 & 2\\ -19 & 27 & 4\end{bmatrix}\right)$

4.
   a) Many examples are possible, e.g. $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$.

   b) $ad-bc=0$; i.e. when the determinant is 0, there is no inverse. Geometrically this is because more than one point will be transformed to the same image point, so there is no longer a one-to-one correspondence between object points and image points, i.e. it is not possible to be certain you are going back to the original point. The zero matrix results in all points in the plane transforming to the origin. Knowing the image is at the origin, it is only possible to say it could have come from any point on the plane.

5.
   a) Many examples are possible.
   b) Some conditions are:
   - The area must be conserved so the determinant of the transformation matrix is 1 or –1.
   - All reflections are their own inverses as two reflections in the same line returns to the initial position.
   - A rotation of 180° is also its own inverse.

c) From CAS, solutions will be of the form $\begin{bmatrix} a & \dfrac{1-a^2}{c} \\ c & -a \end{bmatrix}$, $c \neq 0$.

A special case is when $1-\mathrm{a}^2 = c^2$ and the transformation $\begin{bmatrix} a & \pm\sqrt{1-a^2} \\ \pm\sqrt{1-a^2} & -a \end{bmatrix}$ is a reflection.

If $c = 0$, we get solutions $\begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix}, \begin{bmatrix} \pm 1 & b \\ 0 & \mp 1 \end{bmatrix}$

These solutions include the identity transformation $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and a rotation of 180° about the origin $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

By symmetry $\begin{bmatrix} \pm 1 & 0 \\ c & \mp 1 \end{bmatrix}$ will also be a solution.